Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐☐   Total No. of Pages : 03

Total No. of Questions : 09

B.Tech.(2009-2010 Batches) (Sem.–2)
ENGINEERING MATHEMATICS – II
Subject Code : AM-102
Paper ID : [A0119]

Time : 3 Hrs.   Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. **Write briefly :**

   a) Verify that the following matrix B is orthogonal :

   $$B = \begin{bmatrix} \cos A & 0 & \sin A \\ 0 & 1 & 0 \\ \sin A & 0 & \cos A \end{bmatrix}$$

   b) Show that every square matrix can be written as P+iQ, where P and Q are Hermitian matrices.

   c) Show that the differential equation $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$ is exact.

   d) Solve the differential equation $(D^2 + 6D + 9)y = 0$.

   e) A particle executing simple harmonic motion of amplitude 5 cm has a speed of 8 cm/sec when at a distance of 3 cms from the centre of the path. Find the period of the motion of the particle.

   f) If $\varphi = 3x^2y - y^3z^2$, find *grad* φ at the point (1,–2,–1).

   g) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$, where C is bounded by $y = x$ and $y = x^2$.

   h) Show that the vector field $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ is irrotational.

i) If mean of a Poisson distribution is *m*, then find its S.D.

j) State two applications of χ^2 –test.

SECTION-B

2. a) Reduce the following matrix to normal form and hence find its inverse.   4

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$

b) Test the following system of equations for consistency and solve.   4

$x+2y+z = 3$; $2x+3y+2z = 5$; $3x-5y+5z = 2$; $3x+9y-z = 4$.

3. Find complete solutions of the following differential equations :

a) $(x^3y^2+x)dy + (x^2y^3-y)dx = 0$ 4

b) $y = xp^2 + p$ 4

4. a) Find complementary function and particular integral of the following differential equation :   4

$$(D^2 - 4D + 3)y = \sin 3x \cos 2x$$

b) Find complete solution of the following Legendre differential equation :   4

$$(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$$

5. A constant *e.m.f E* at time $t = 0$ is applied to circuit consisting of inductance *L*, resistance *R* and capacitance *C* in series. The initial values of the current and the charge being zero, find the current at any time *t*, if $CR^2 < 4L$.   8

SECTION-C

6. a) Find the directional derivative of $\varphi(x,y,z) = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.   4

b) If $\vec{F} = 2y\hat{i} - z\hat{j} + x\hat{k}$, evaluate $\int_C \vec{F} \times d\vec{R}$ along the curve $C : x = \cos t$, $y = \sin t$, $z = 2 \cos t$ from $t = 0$ to $t = \pi/2$.   4

7. a) Apply Green's theorem, evaluate $\int_C [(y - \sin x)dx + \cos x\, dy]$, where C is the plane

   triangle enclosed by the lines $y = 0$, $x = \pi/2$ and $y = \dfrac{2}{\pi}x$.   4

   b) Verify Stoke's theorem for a vector field defined by $\vec{F} = -y^3\hat{i} + x^3\hat{j}$ in the region $x^2 + y^2 \leq 1$, $z = 0$.   4

8. a) The probability density function of a variate X is given below :

X :	0	1	2	3	4	5	6
p(X) :	k	3k	5k	7k	9k	11k	13k

   Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$. What will be the minimum value of k so that $P(X \leq 2) > 3$?   4

   b) Fit a second degree parabola to the following data :   4

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

9. a) In two large populations there are 30% and 25% respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations?   4

   b) In an experiment on pea breeding, the following frequency of seeds were obtained :

Round & Yellow	Wrinkled & Yellow	Round & Green	Wrinkled & Green	Total
315	101	108	32	556

   Theory predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment.   4